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Note Title 7/13/2010

List $\{1, 2, 2, 5\}$ $\quad \mu_x = \dfrac{1+2+2+5}{4}$

$$= \frac{1+(2)2+5}{4} = 1\left(\tfrac{1}{4}\right)+2\left(\tfrac{2}{4}\right)+5\left(\tfrac{1}{4}\right)$$

SUM OF VALUES, SOME OF WHICH ARE DUPLICATED CAN BE REARRANGED INTO A WEIGHTED SUM.

$$\mu = \sum x / n = \sum v_i \#_i / n$$

DISTINCT VALUES

$$= \sum v_i \cdot w_i$$

OF ITEMS ON LIST X TAKING VALUE v_i

SO $v_1 = 1$, $v_2 = 2$, $v_3 = 5$,

$w_1 = 1/4$, $w_2 = 2/4$, $w_3 = 1/4$

$\sum w_i = 1 \quad w_i \geq 0$ REFER TO SUCH WEIGHTS AS PROBABILITIES.

WHAT ABOUT σ_x^2? RECALL $\sigma^2 = \mu_{x^2} - \mu_x^2$

ONLY DEPENDS ON v_i w_i

DISTRIBUTION OF A LIST.

REFER TO THE DISTRIBUTION AS

- DISTINCT VALUES v_i ON LIST
- RELATIVE WEIGHTS w_i (PROBABILITIES)

μ, σ^2 DEPEND ONLY ON THE DISTRIBUTION.

EXPECTATION (MEAN) AS AN OPERATION PERFORMED ON A DISTRIBUTION.

$$\{v_i\}, \{p_i\} \qquad p_i \geq 0 \qquad \sum p_i = 1$$

DISTINCT NUMERICAL VALUES

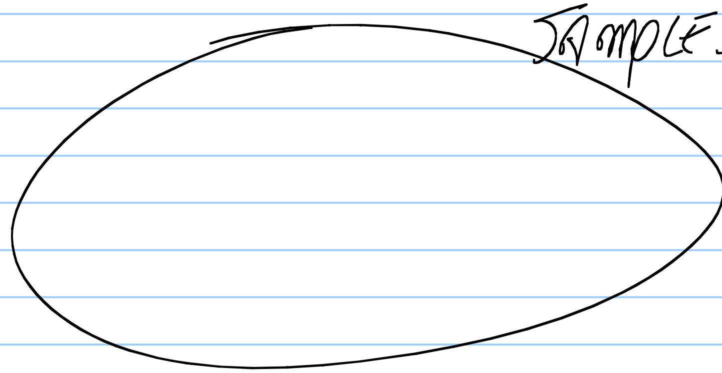
PROBABILITIES FOR THEM

$$\mu = \sum_i \text{value} \cdot \text{pr.} = \sum v_i p_i$$

$$\sigma^2 = \sum_i (v_i - \mu)^2 p_i$$

MIGHT HAVE NEED TO THINK IN TERMS OF THE DISTRIBUTION AS ARISING FROM A PROBABILITY EXPERIMENT.

SAMPLE SPACE



e.g. BAG $\lfloor 3R \ 4G \ 2Y \rfloor$

DRAW TWICE WITHOUT REPLACEMENT

SCORE $X = 4 \#G + 2(\#Y)^2$

PROBABILITY		x						
$\frac{3}{9}\frac{2}{8}$	$R_1 R_2$	$= 0$	$G_1 R_2 = 4$	$\frac{4}{9}\frac{3}{8}$	$Y_1 R_2 = 2$	$\frac{2}{9}\frac{3}{8}$		
$\frac{3}{9}\frac{4}{8}$	$R_1 G_2$	$= 4$	$G_1 G_2 = 8$	$\frac{4}{9}\frac{3}{8}$	$Y_1 G_2 = 6$	$\frac{2}{9}\frac{4}{8}$		TOTAL
$\frac{3}{9}\frac{2}{8}$	$R_1 Y_2$	$= 2$	$G_1 Y_2 = 6$	$\frac{4}{9}\frac{2}{8}$	$Y_1 Y_2 = 8$	$\frac{2}{9}\frac{1}{8}$		PROBY $= 1$

$EX = \sum$ values x times respective probabilities

$= 0 \frac{3}{9}\frac{2}{8} + 4 \frac{3}{9}\frac{4}{8} + \cdots + 8 \frac{2}{9}\frac{1}{8}$

INSTRUCTION TO COMPUTE THE PROBABILITY WEIGHTED AVG.

$= EX =$ EXPECTED VALUE OF RANDOM VARIABLE X.

RANDOM VARIABLE X: NUMERICAL ASSIGNMENT OF VALUES x TO OUTCOMES OF A PROBABILITY EXPERIMENT.

PROBABILITY DISTRIBUTION OF RANDOM VARIABLE (R.V.) X:

0	2	4	6	8	VALUES
3/9 2/8	3/9 2/8	4/9 3/8			2/9 1/8	PROBABILITIES.

PROPERTIES OF E

$$E(X+Y) = EX + EY$$

$\boxed{3R\ 4G\ 2Y}$

$$E(aX + bY + c) = aEX + bEY + c$$

RECALL X ABOVE WAS $X = 4(\#G) + 2(\#Y)^2$

THEREFORE $EX = 4\,E(\#G) + 2\,E(\#Y)^2$

? $E(\#G)$

LET $G^{(1)} = \begin{cases} 1 & G_1 \\ 0 & G_1^c \end{cases}$ $\quad EG^{(1)} = 1(4/9) + 0(5/9)$

$= 4/9$

COMPONENTS OF X

$G^{(2)} = \begin{cases} 1 & G_2 \\ 0 & G_2^c \end{cases}$ SAME $EG^{(2)} = 4/9$

SAME AS DRAW ONE

$Y_1^c Y_2 ;\ \frac{2}{9}\frac{7}{8}$

$Y_1 Y_2^c$

$2\left(\frac{2}{9}\right)\frac{7}{8}$

So $E(\#G) = E\left(G^{(1)} + G^{(2)}\right)$
$= 2\left(4/9\right) = 8/9$

$E(\#Y)^2 = 0^2\, \frac{7}{9}\frac{6}{8} + 1^2\, \frac{2}{9}\frac{7}{8}$
$+ 2^2\, \frac{2}{9}\frac{1}{8}$
$= \frac{2}{9}\frac{7}{8} + \cancel{8}\, \frac{2}{9}\, \frac{1}{\cancel{8}}$

$Y_1 Y_2 \quad Y_1 Y_2^c$

$Y_1^c Y_2 \quad Y_1^c Y_2^c$

$\#Y =$	0	1	2
	$\frac{7}{9}\frac{6}{8}$	$2\,\frac{2}{9}\frac{7}{8}$	$\frac{2}{9}\frac{1}{8}$

RECALC $X = 4(\#G) + 2(\#Y)^2$

$EX = 4\,E(\#G) + 2\,E(\#Y)^2$
$= 4\left(8/9\right) + 2\left(\frac{2}{9}\frac{7}{8} + 2/9\right)$

NOTE:

$\#G$	0	1	2
	$\frac{5}{9}\frac{4}{8}$	$\frac{40}{9\,8}$	$\frac{4}{9}\frac{3}{8}$

3R 4G
2Y

$$E(\#G) = 0\,\frac{5}{9}\frac{4}{8} + 1\,\frac{40}{9\,8} + 2\,\frac{4}{9}\frac{3}{8}$$

? $EG^{(1)} + EG^{(2)} = 8/9$

SAME!!

TAKE-AWAYS

EX IS THE PROBABILITY WEIGHTED AVERAGE (MEAN) RETURN FROM RANDOM VARIABLE X.

CAN GET $EX = \sum x\, p(x)$ FIRST THING WE DID.

ALSO, $EX = \sum v \, p(v)$ ← DISTINCT VALUES OF X

JUST COLLECTING TERMS IN THE FIRST CALCULATION OF EX JUST ABOVE.

a, b, c CONSTANTS –

RULES OF E: $E(aX + bY + c)$

$= aEX + bEY + c$

LIKE "AVG \$ HELD IS SUM OF AVG COIN \$ + AVG PAPER \$".

EXAMPLE OF THIS? $\boxed{3R \ 4G \ 2Y}$

2 DRAWS WITHOUT REPLACEMENT.

DEFINE $X = 4(\#G) + 2(\#Y)^2$

EX = YOUR AVG RETURN ON ONE PLAY.

CAN GET $EX = 4E(\#G) + 2\,E(\#Y)^2$

$\#G = G^{(1)} + G^{(2)}$

$E\#G = (4/9 + 4/9)$

DIST OF #Y

DIST OF $(\#Y)^2$

$E(\#Y)^2$

GET

EX

DIST OF #G

0	1	2
$\frac{5}{9}\frac{4}{8}$	$\frac{40}{9\cdot 8}$	$\left(\frac{4}{9}\right)\left(\frac{3}{8}\right)$

SAME AS

EXAMPLE 1. 100 TOSSES OF DIE X = SUM OF 100 TOSSES.

$$EX = E(X_1 + \cdots + X_{100})$$

$$= EX_1 + EX_2 + \cdot + EX_{100}$$

$$= 3.5 \qquad 3.5 \qquad 3.5 \qquad = 350$$

EXAMPLE 2. TOSS COIN 100 TIMES.

$$X = \#H = \underset{1/2}{\begin{matrix} 1 & H_1 \\ 0 & T_1 \end{matrix}} + \underset{1/2}{\begin{matrix} 1 & H_2 \\ 0 & T_2 \end{matrix}} + \cdots + \underset{1/2}{\begin{matrix} 1 & H_{100} \\ 0 & T_{100} \end{matrix}}$$

FORMULA: $EX = np$

BINOMIAL.

$$\operatorname{Var} X \underset{\text{TOSSES}}{\overset{\text{INDEP}}{=}} \operatorname{Var} \begin{matrix} 1 & H_1 \\ 0 & T_1 \end{matrix} + \cdots + \operatorname{Var} \begin{matrix} 1 & H_{100} \\ 0 & T_{100} \end{matrix}$$

$$= pq + \cdots + pq = npq.$$ VARIANCE OF BINOMIAL.